Unit-04 Chapter-04

Moving charges and magnetism (12Hours)

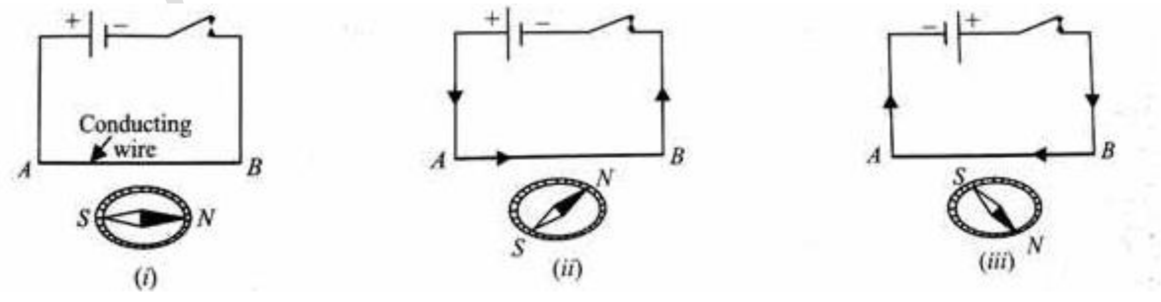
Concept of magnetic field - Oersted's experiment - Force on a moving charge in uniform magnetic and electric fields: Lorentz force - Derivation of magnetic force on a current carrying conductor $\vec{F} = I\,(\vec{l} \times \vec{B})$.
Motion of a charge in a uniform magnetic field: Nature of trajectories - Derivation of radius and angular frequency of circular motion of a charge in uniform magnetic field.
Velocity selector: Crossed electric and magnetic fields serve as velocity selector. Cyclotron: Principle, construction, working and uses.
Biot-Savart law: Statement, explanation and expression in vector form - Derivation of magnetic field on the axis of a circular current loop - Right hand thumb rule to find direction.
Ampere's circuital law: Statement and explanation - Application of Ampere's circuital law to derive the magnetic field due to an infinitely long straight current carrying wire: Solenoid and toroid - Mention of expressions for the magnetic field at a point inside a solenoid and a toroid.
Derivation of the force between two parallel current carrying conductors - Definition of ampere.
Current loop as a magnetic dipole - Qualitative explanation and definition of magnetic dipole moment -Mention of expression for torque experienced by a current loop in a magnetic field - Derivation of magnetic dipole moment of a revolving electron in a hydrogen atom and to obtain the value of Bohr magneton.
Moving coil galvanometer: Mention of expression for angular deflection - Definitions of current sensitivity and voltage sensitivity - Conversion of galvanometer to ammeter and voltmeter, **Numerical Problems**.

- **Introduction:**

  In olden days, electricity and magnetism were treated as separate subjects. In 1820, Danish physicist Hans christen Oerstead demonstrated that both electricity and magnetism are intimately related to each other. This leads to unification of electricity and magnetism.

- **Oerstead`s Experiment:**

  Oersted experiments showed that the electric and magnetic phenomena are related to each other.



1. Fig1 shows a conducting wire AB above a magnetic needle parallel to it. So long as there is no current in the wire, the magnetic needle remains parallel to the wire i.e there is no deflection in the magnetic needle.
2. As soon as the current flow through the wire AB (fig b), the needle is deflected. When the current in wire AB is reversed (fig c), the needle is deflected in the opposite direction. This field around a current-carrying conductor.
3. On increasing the current in the wire AB, the deflection of the needle is increased and vice—versa. This shows that magnetic field strength increases with increases in current and vice-versa.

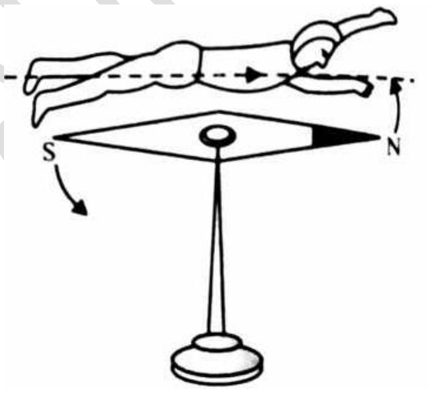
It is clear from oersted experiment that a current carrying conductor produced a magnetic field around it. The larger the value of current in the conductor, the strength of magnetic field and vice- versa.

- **Ampere`s swimming rule:**

The direction of deflection of the magnetic needle due to current in the wire is given by ampere`s swimming rule.

Statement:

Imagine a man swimming along the wire in the direction of the low of current with his face always turned towards the magnetic needle so that the current enters through his feet and leaves at his head. Then N-pole of the magnetic needle will be deflected towards his left hand.

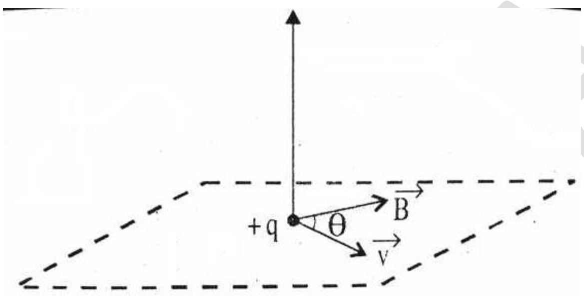


- **Magnetic field:**

The space around a current carrying conductor where magnetic effect can be experienced is called a magnetic field is called magnetic field strength or magnetic induction or magnetic flux density.

- **Note:**
    1. Magnetic field is a vector quantity.
    2. The unit of magnetic field is Tesla or wb/m^2.
    3. The magnetic flux density is a measure of field concentration i.e. number of lines in each square meter of the field. It is, therefore, the appropriate quantity to describe the magnetic field.
    4. Just as electric field around a charge is described by the field vector $\overrightarrow{E}$, similarly magnetic field around a current carrying conductor is described by the magnetic field vector $\vec{B}$.
- **Magnetic force:**

Consider a positive charge q moving with velocity $\overrightarrow{V}$, in the presence of magnetic field $\vec{B}$.it is experimentally found that the magnetic force on the charge q is given by $\vec{F}$ =q($\vec{V}$x$\vec{B}$)----1



The magnetic force $\vec{F}$ on the moving charge has the following features.

1. The magnetic force $\vec{F}$ depends on the charge q. velocity$\vec{V}$. And the magnetic field$\vec{B}$. Force on a –ve charge is opposite to that on +ve charge.
2. The force becomes zero if velocity and magnetic field are parallel to each other. It is maximum if velocity and magnetic field are perpendicular to each other.
3. The magnitude of the force is zero if the charge is not moving (v=0). Only moving charge experiences force in a magnetic field.
4. The direction of force is always perpendicular to both velocity and magnetic field. The direction of the force is given by right hand rule **or** the right handed screw rule for vector product or cross product.

∴ The magnitude of the force on the charge q moving with a velocity $\vec{V}$, in the magnetic field $\overrightarrow{B}$ is $\vec{F}$ =q($\vec{V}$x$\vec{B}$)=qVBsinɵ.

Where ɵ-angle between the direction of $\vec{V}$x$\vec{B}$

We have F=B when q=1unit V=1unit and ɵ=90^0.

Hence, magnetic field is the force experienced by unit +ve charge moving with a unit velocity perpendicular to the direction of magnetic field.

- **One Tesla :**

The magnetic field strength is said to be 1 tesla. If 1 coulomb of charge moving with a velocity of 1 m/s, perpendicular to the magnetic field experiences a force of 1N.

i.e B=1T if q=1C V=1m/s $\theta=90^0$. And F=1N.

- **Note:**

1. Another unit used for magnetic field is gauss(1gauss=10^{-4} tesla)
2. Dimensional formula of magnetic field is $[MA^{-1}T^{-2}]$

- **Lorentz force:**

If the +ve charge q is moving with velocity$\vec{V}$, in the presence of both electric field $\vec{E}$ and magnetic field $\vec{B}$. The force on the charge due to both of them is

Lorentz force= electric force + magnetic force

$\vec{F}$=q [$\vec{E}$+($\vec{V}$x$\vec{B}$)]

Def: **Lorentz force is the force experienced by the charged particle moving in a region where both magnetic field and electric field are present.**

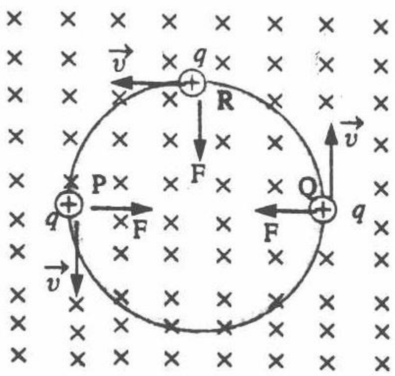
- **Charged particle moving in a uniform magnetic field (motion in a magnetic field)**

When a charged particle of charge +q and mass m moving with a velocity $\vec{V}$, enters a uniform magnetic field $\vec{B}$.it experiences a magnetic force $\overrightarrow{F_m}$ is given by $\overrightarrow{F_m}$ = q($\vec{V}$x$\vec{B}$).

Clearly, the magnetic force acts right angles to the planning containing $\vec{V}$ and $\vec{B}$. The magnitude of this force is F_m=B qVsinθ where θ- angle between $\vec{V}$ and $\vec{B}$.

I) **A moving charged particle does not experience any force**. If it motion is parallel or antiparallel to the magnetic field. Therefore the particle will continuous to move in the original direction [straight path]

II) When charged particle moves at right angle to the magnetic field.



Here the charged particle (+q) enters the magnetic field in the plane of the paper and magnetic field is perpendicular to the plane of the paper directed outward. The magnetic field Fm is always perpendicular to the direction of motion of the charged particle; it only changes the direction of motion of the charged particle. As a result.

1. **Charged particle moves in a circle of radius r perpendicular to field.**

   In fact, magnetic force (Fm=qVB) provides the necessary centripetal force(Fc=$\frac{MV^2}{r}$)

   $\therefore \frac{MV^2}{r}$=qVB or radius of path. $r = \frac{MV}{qB}$.

   For a given charge mass and magnetic field rαV. This means that fast particles moves in large circle and slow ones in small circles.

2. **The time period of revolution of the charged particle in the magnetic field is**

   Time =$\frac{Distance}{speed}$=$\frac{circumference\ of\ the\ circle}{speed}$

   T=$\frac{2\pi r}{V}$ but $r = \frac{MV}{qB}$.

   T=$\frac{2\pi M}{qB}$

3. **Frequency of the charged particle**

   ν=$\frac{qB}{2\pi M}$

   Thus period and frequency is independent of speed of particle and radius of the circular path. It dependes on the magnetic field B and the nature of the particle.

   This fact is used to accelerate charge particle in a cyclotron.

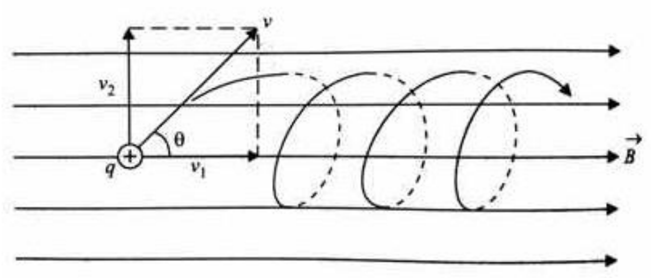
4. **Angular frequency of the charged particle.**

   ω= 2πν=2πx$\frac{qB}{2\pi M}$=$\frac{qB}{M}$

   It is clear that angular frequency does not depends upon the speed of particle

Thus all the charged particles take the same time to complete the circular orbit of smaller large radius. Provides their specific charge (q/m) is same.

III) When the charged particle moves at an angle to the magnetic field (other than 0^0,90^0 and 180^0.)



Suppose the charged particle moving with velocity $\vec{V}$ enters a uniform magnetic field$\vec{B}$. Making an angle ɵ to the direction of the field as shown in figure.

The velocity $\overrightarrow{V}$ can be resolved into rectangular components

a). $V\cos\theta = V_1$ acting in the direction of the field.

b). $v\sin\theta = V_2$ acting perpendicular to the direction of the field.

The charged particle moves with constant velocity $V\cos\theta$ along the magnetic field as no force acts on the charged particle when it moves parallel to the magnetic field.

Since $V\sin\theta$ is perpendicular to the direction of $\vec{B}$, so the particle experiences a force. Under this force charged particle tends to moves in a circular path in a plane perpendicular to the magnetic field. Consequently the charged particle will follow a helix path.

Centripetal force required moving the particle in a circle is provided by the magnetic Lorentz force

i.e $\frac{M(V\sin\theta)^2}{r} = q\,(v\sin\theta)\,B\sin\theta$. $\quad$ $\sin\theta = 1\ (\theta = 90^0.)$

$\frac{M(V\sin\theta)}{r} = qB$

$r = \frac{M(V\sin\theta)}{qB}$-------radius of the circular path.

Time period of particle $T = \frac{2\pi r}{V\sin\theta}$ but $r = \frac{MV\sin\theta}{qB}$. $\quad T = \frac{2\pi M}{qB}$

Frequency of the charged $\quad \nu = \frac{qB}{2\pi M}$

Angular frequency of the charged particle. $\omega = 2\pi\nu = 2\pi \times \frac{qB}{2\pi M} = \frac{qB}{M}$

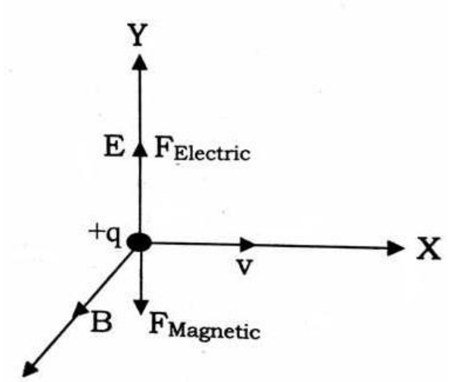
Pitch of helix: the linear distance travelled by the charged particle in one attention is called pitch of the helix.

i.e pitch of the helix $P = V\cos\theta \times T = V\cos\theta \times \frac{2\pi M}{qB}$

$P = \frac{2\pi MV\cos\theta}{qB}$

- **Velocity selector (velocity filter)**

It is an arrangement of crossed uniform electric and magnetic fields used to select the charged particles of particular velocity out of beam of charged particles having different velocities.



Consider a charged particle q is entering a crossed field with velocity V in the direction of perpendicular to both of them as shown in figure.

Now it experiences both electric force Fe and magnetic force Fm

$F_e=qE$ $F_m=BqV\sin\theta$ and these two acting in perpendicular directions.

If value of E and B are adjusted such that Fe=Fm

$qE=BqV(\theta=90^0.)$ **$V=\frac{E}{B}$.**

- **Uses of velocity selector:**
    1. Velocity selector used to measured specific charge of electrons
    2. Velocity selector is used as principle of mass spectroscope.(a device used to separate ions)
- **Note:**

Crossed field: the two uniform electric and magnetic fields which are mutually perpendicular to each other are called crossed fields.

Only the particles with speed equal to E/B pass undeflected through the region of crossed fields.

- **Bio-Savart Law:**

In 1819 Orested discovered the magnetic effect of current. He observed that a magnetic needle was deflected by a current carrying conductor. Later Biot and Savart arrived at an expression for magnetic field at a point due to current element.

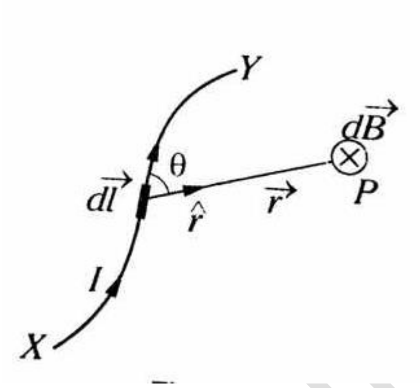
Statement:

The strength of the magnetic field (dB) at a point due to a current element is

1. Directly proportional to the current (I).
2. Directly proportional to the length of current element (dl).

3. Directly proportional to the sine of the angle between the element and the line joining the point to the element($\sin\theta$)
4. Inversely proportional to square of the distance between the point and the current element (r^2).

Explanation:



Consider a conductor XY carrying current I. Let AB be a current element of length dl.

Let P be a point at a distance r from the current element.

According to Biot- Savart law the magnetic field dB at P is.

dB α $\frac{Idl\sin\theta}{r^2}$ or dB =K $\frac{Idl\sin\theta}{r^2}$ where K-constant. Its value depends on medium between the element and the point, the system of unit chosen.

K=$\frac{\mu_o}{4\pi}$ μ_0-permeability of free space =$4\pi x10^{-7}Hm^{-1}$.

$\therefore dB=\frac{\mu_o}{4\pi}\frac{Idl\sin\theta}{r^2}$.

Vector form:

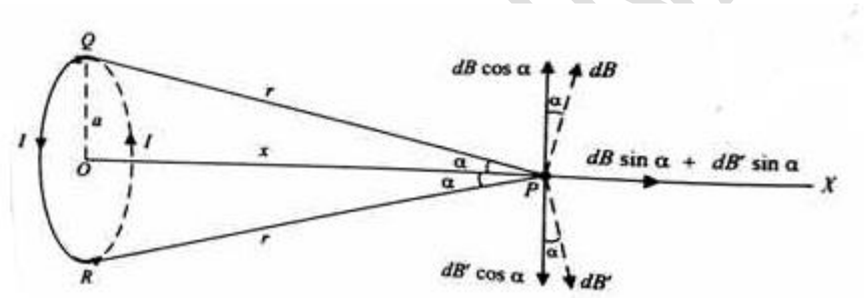
If we consider length of element as $\vec{dl}$. Distance of P as displacement vector $\vec{r}$ and unit vector as $\hat{r}$ then,

$\vec{dB}=\frac{\mu_o}{4\pi}\frac{I\vec{dl}x\hat{r}}{r^2}$ now $\vec{r}=|\vec{r}|\hat{r}$ $\therefore \hat{r}=\frac{\vec{r}}{|\vec{r}|}$.

$\vec{dB}=\frac{\mu_o}{4\pi}\frac{I\vec{dl}x\vec{r}}{r^3}$

Current element: a small portion of any conductor carrying current is known as current element.

- **Comparison between Biot- savarts law for magnetic field and coulombs law for electric field.**
    1. Both are long range, since both depends inversely on the square of distance from the source to the point of intersect.
    2. The principle of superposition applies to both fields.
    3. Magnetic field produced by a vector source i.e. current element. And electric field is produced by a scalar source i.e. electric charge.
    4. Biot –Savart law is angle dependent and coulomb's law is angle independent.
    5. Biot-Savart law takes into account the permeability of free space(μ_0) and coulomb's law into the account of free permittivity of free space(ϵ_0).
- **Magnetic field on the axis of a circular current loop**



Consider a circular coil of radius a. Carrying a current I in the direction as shown in figure.

Let the plane of the coil be perpendicular to the plane of the paper. It is desired to find the magnetic field at a point P on the axis of the coil such that OP=x

Consider two small current element each of length dl located diametrically opposite to each other at Q and R .suppose the distance of Q and R from P is r. i.e PQ=PR=r

∴ r=$\sqrt{a^2+x^2}$ let ∠QPD=∠RPQ= α

According to Biot-Savart Law magnetic field at Q is

dB=$\frac{\mu_o}{4\pi}\frac{Idlsin\theta}{r^2}$ (θ=90 sin90=1)

dB=$\frac{\mu_o}{4\pi}\frac{Idl}{a^2+x^2}$

Similarly magnetic field at R is

dB=$\frac{\mu_o}{4\pi}\ \frac{Idl}{a^2+x^2}$

The magnetic field dB is resolved into two components along X and Y axis similarly magnetic field dB` is resolved into two components as shown in figure.

It is clear that vertical component (dBcosα and Bcosα) will be equal and opposite and thus cancel each other. However component along the axis of the coil (dBsinα and dB`sinα) are added and act in direction PX.

Hence resultant magnetic field at point P is the vector sum of all the component dBcosα over the entire coil.

B=$\int dBsin\alpha$.

B=$\int \frac{\mu_o}{4\pi}\ \frac{Idlsin\alpha}{a^2+x^2}$

B=$\frac{\mu_o}{4\pi}\ \frac{Isin\alpha}{a^2+x^2}\ \int dl$

Now sinα = $\frac{a}{r}$=$\frac{a}{\sqrt{a^2+x^2}}$. And $\int dl$=2πa

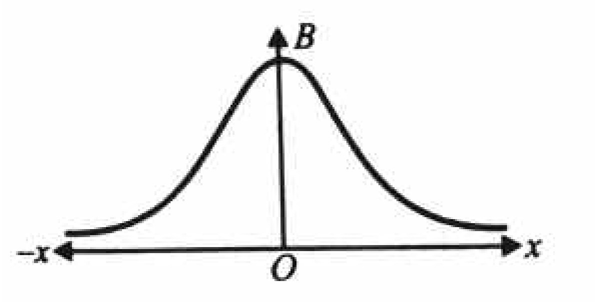
∴ B=$\frac{\mu_o}{4\pi}\ \frac{Ia^2}{(a^2+x^2)^{\frac{3}{2}}}$ along PX

If the circular coil has n turns then

B=$\frac{\mu_o}{4\pi}\ \frac{n\,Ia^2}{(a^2+x^2)^{\frac{3}{2}}}$ along PX

Cases:

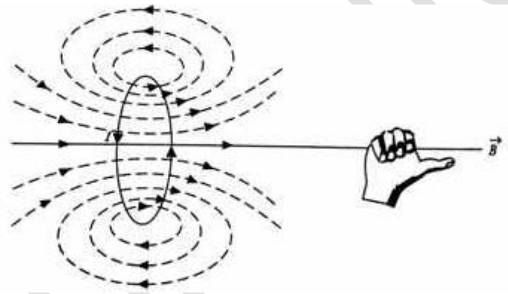
1. When a point P is at the centre of the coil
   In that case X=0 ∴B= $\frac{n\,I\mu_o}{2a}$
2. When a point P is on the axial line far away from the centre of coil in that coil X>>a so that a^2+x^2=x^2
   B=$\frac{\mu_0 nIa^2}{2x^3}$
3. The value of magnetic field B is maximum at the centre of the coil however, the value of B decreases as we go away from the centre on either side of the coil.



- **Right hand thumb rule:**

According to this rule curl the palm of your and around the circular wire with the fingers pointing in the direction of the current, the right hand thumb gives the direction of the magnetic field. Or

Imagine that you hold the axis of the coil in the right hand fist in such a way that fingers points in the direction of current in the coil. Then outstretched thumb gives the direction of magnetic field.



- **Ampere`s circuital law:**

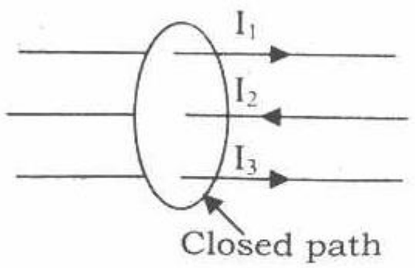
Just as gauss law is an alternate form of coulomb's law in electrostatics. Similarly we have ampere`s circuital law as an alternative form of Biot-savart in magnetism. Ampere circuital law gives the general relation between a current in a wire of any shape and the magnetic field produced around it.

Statement :

The line integral of magnetic field $\vec{B}$ around any closed path in vacuum /air is equal to μ_o times the total current enclosed by the path

i.e $\oint B.dl=\mu_0$ I

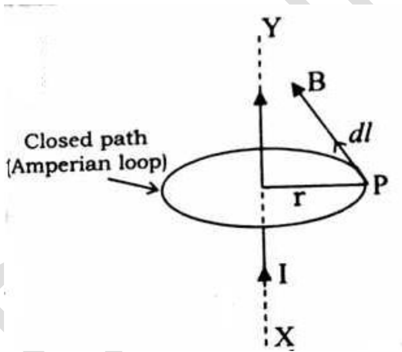
Ex:



in the figure I_1,I_2 and I_3 are the current enclosed by the closed path. Here I_2 is opposite to I_1 and I_3.

$\oint B.dl = \mu_0 (I_1 - I_2 + I_3)$

- **Limitations of ampere`s circuital law:**
    1. Ampere`s circuital law is not universal law.
    2. Ampere`s circuital law deals with steady current only.
- **Magnetic field strength due to a straight current carrying wire:**



Consider a straight conductor XY carrying current I.

Let B is the magnetic field at P. dl-is the line element of ampere`s loop. Draw a circle of radius r such that the wire is perpendicular to the plane of the circle passing through its centre. The direction of B is tangential to the circle at any point on the circle.

From ampere`s circuital law

$\oint B.dl = \mu_0\, x$ (net current enclosed by the path)

$\oint B.dl = \mu_0\, xI$-----------1

But

$\oint B.dl = \oint B.dl\cos\theta$

$\oint B.dl = \oint B.dl$

$\oint B.dl$= $B\oint dl$

$\oint B.dl$= Bx2πr--------2

On comparing 1 and 2 we get

Bx2πr=$\mu_0 I$

B=$\frac{\mu_0 I}{2\pi r}$

For this result we have the following points of view

1. The magnitude of the magnetic field at every point on a circle of radius r is the same.

2. The direction of the magnetic field at every point on the circle is tangential to it. The lines of constant magnetic field from concentric lines. These lines are called on field lines. Thus magnetic field lines from closed path without starting and ending point.

3. The expression for magnetic field due to current in a straight wire provides theoretical support to Oersterd experiments.

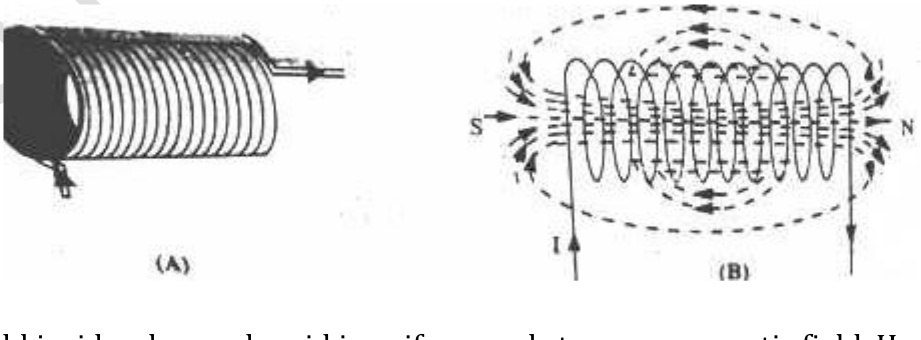
- ***The solenoid and the Toroid:***

The solenoid and Toroid are two pieces of equipment which generate magnetic field.

- ***Solenoid:***

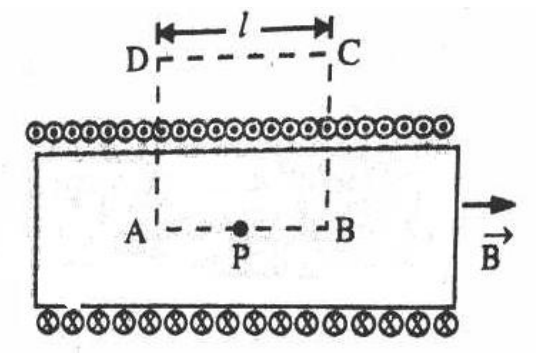
It is a cylindrical coil made of a large number of turns of insulated conducting wire.

A solenoid has coated wire wound in the form of helix. Each turns of the solenoid can be regarded as a circular loop. Total magnetic field due to the whole solenoid will be equal to the vector sum of the magnetic field of each turn. A long solenoid looks like a hollow pipe.



The magnetic field inside a long solenoid is uniform and stronger magnetic field. However magnetic field at a point outside the long solenoid is non-uniform and weak magnetic field. For an ideal solenoid the magnetic field at a point outside the solenoid is practically zero.

- ***Magnetic field due to a current carrying solenoid.***



Consider a very long solenoid having **n** turns per unit length of solenoid.

Let current I be flowing through the solenoid. Let P be point well within the solenoid. Consider any rectangular loop **ABCD** (known as Amperian loop) passing through **P** as shown in figure.

Then $\oint \vec{B}.\vec{dl}$ line integral of magnetic field across the loop ABCD

$$=\int_A^B \vec{B}.\vec{dl}+\int_B^C \vec{B}.\vec{dl}+\int_C^D \vec{B}.\vec{dl}+\int_A^D \vec{B}.\vec{dl}\text{---------}1$$

$\vec{B}$ Is perpendicular to paths BC and AD i.e. angle between $\vec{B}$ &$\vec{dl}$ is 90^0 for these paths.

$\therefore \int_B^C \vec{B}.\vec{dl} = \int_A^D \vec{B}.\vec{dl} = \int B dl \cos 90 = 0$

Since path CD is outside, where $\vec{B}$ is taken as zero, so $\int_C^D \vec{B}.\vec{dl}=0$

For path AB, the direction of $\vec{dl}$ and $\vec{B}$ is same i.e. ɵ=0 hence equation 1 becomes

$\oint \vec{B}.\vec{dl} = \int_A^B \vec{B}.\vec{dl} = \int_A^B B dl \cos 0 = \int_A^B B dl$

$\oint \vec{B}.\vec{dl} = B\int_A^B dl$

$\oint \vec{B}.\vec{dl} = Bl$**----------2**

According to ampere`s circuital law

$\oint \vec{B}.\vec{dl}=\mu_0$**x net current enclosed by loop ABCD**

$\oint \vec{B}.\vec{dl}=\mu_0$**x number of turns in the loop ABCD x I=μ_0nIl------3 comparing equation 2 and 3**

B l =μ_0nIl

B=μ_0nI

Thus magnetic field well within an infinitely long solenoid is given by

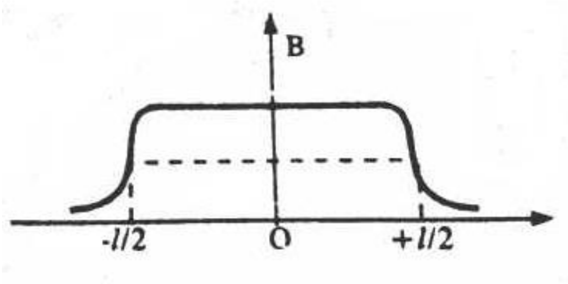
$B=\mu_0 n I$

Since n=N/l, where N=total number of turns of solenoid.

$B=\mu_0 \frac{N}{l} I$

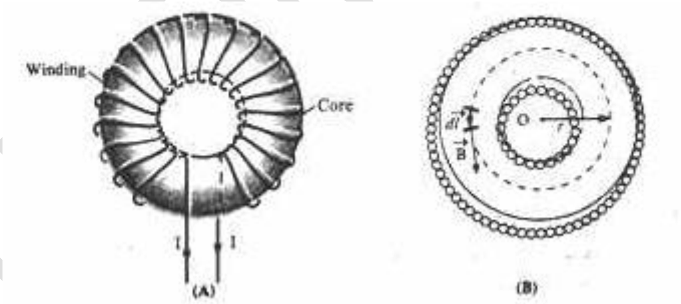
- **Note:**

The variation of magnetic field of the solenoid with distance from the centre of the solenoid and along the axis of solenoid as shown in figure.



- **The Toroid:**

A Toroid can be considered as a ring shaped closed solenoid. A toroid is a hollow circular ring of finite thickness on which a large number of insulated wires are closely wound.



Consider a toroid having **n** turns per unit length. Let **I** be the current flowing through the toroid. The magnetic field lines of force mainly remain in the core of toroid and are in the form of concentric circle. Consider a circle of radius **r**. (above figure).

$\oint \vec{B}.\vec{dl}=\int Bdl\cos\theta$

By symmetry, magnetic field $\vec{B}$. in the coilis constant and is along the tangent to path $\vec{dl}$. Therefore, angle θ between $\vec{B}$. and $\vec{dl}$ is 0. Hence

$\oint \vec{B}.\vec{dl}=\int Bdl\cos 0$

$\oint \vec{B}.\vec{dl} = \int Bdl$

$\oint \vec{B}.\vec{dl} = B\oint dl$

$\oint \vec{B}.\vec{dl}$=**B x circumference of circle of radius r**

$\oint \vec{B}.\vec{dl}$=**Bx 2πr----------1**

According to ampere`s circuital law

$\oint \vec{B}.\vec{dl} = \mu_0$**x net current enclosed by the circle of radius r.**

$\oint \vec{B}.\vec{dl} = \mu_0$**x number of turns xI=**$\mu_0 nI2\pi r$**------3**

Comparing equation 1 and 2

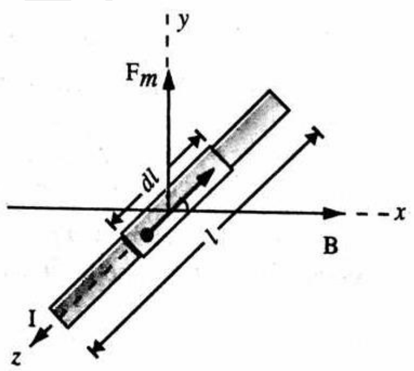
$B2\pi r\text{-}=\mu_0 nI2\pi r$

$B=\mu_0 nI$

Since n=N/2πr, where N=total number of turns of toroid .

$B=\mu_0 \frac{N}{2\pi r}I$

- **Expression for the force experienced by a current carrying conductor placed in uniform magnetic field.**



Consider a conductor of length l and area of cross-section A placed at an angle ɵ to the direction of a uniform magnetic field of intensity $\vec{B}$. As shown in figure

Let I-be the study current through conductor. n- be the number of free electron per unit volume of the conductor

Then total number of free electron in the conductor = n x volume of the conductor q= nx Alxe   where e-charge of electron

The magnetic force on the conductor is F=BqV sinθ

If V_d is the drift velocity of the free electrons

F=BqVd sinθ   but   q=nAle

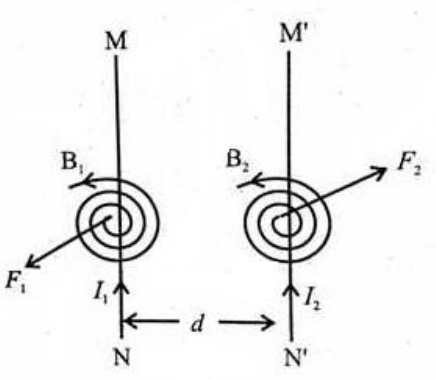
$F=BnAleV_d$ sinθ $I=nAV_de$

F=BIlsinθ or F=I $(\vec{l} \times \vec{B})$

Case1. If θ=0^0 force F=0 no force acting

Case2. If θ=90^0 force F=BIl maximum force is acting.

- **Force between two straight parallel current carrying wire and hence define ampere.**



Consider two parallel wires **X** and **Y** carrying currents **I_1** and **I_2** respectively in the same direction and producing magnetic field **B_1** and **B_2**. Let l be the length of each conductor and **d** is the separation between them.

Magnetic field due to current **I_1** is given by

$\boldsymbol{B_1=\frac{\mu_0 I_1}{2\pi d}}$ **-------1**

According to right hand grasp rule, the direction of $\boldsymbol{B_1}$ is perpendicular to the plane of the paper and directed inwards. The wire Y lies in the magnetic field $\boldsymbol{B_1}$ and hence it experience force.

The force on a segment **l** of conductor **Y** is given by

$\boldsymbol{F_1=B_1 I_2 l sin}\theta$

$F_1 = B_1 I_2 l$--------2   ($\sin\theta = 1$ or $\theta = 90^0$)

From equation 1 and 2

$F_1 = \frac{\mu_0 I_1}{2\pi d} I_2 l$. Similarly $F_2 = \frac{\mu_0 I_2}{2\pi d} I_1 l$

According to Fleming's left hand rule, F_1 is towards the wire X and F_2 is towards the conductor Y. i.e. X and Y are attracting each other. When the currents are in the opposite direction, X and Y repel each other. F_1 and F_2 are the mutual forces. Form Newton's third law $F_1 = -F_2$

Therefore, the force of attraction or the force of repulsion is given by,

$F = \frac{\mu_0 I_1 I_2 l}{2\pi d}$

- **Define ampere using above relation.**

Ampere is the current which when flows through two infinitely long straight parallel wires separated by a distance 1 meter, placed in free space, causes a force of 2×10^{-7} Nm^{-1} between them.

To fine ampere.

Consider $\frac{F}{l} = \frac{\mu_0 I_1 I_2}{2\pi d}$

$$\frac{F}{l} = \frac{4\pi x 10^{-7} I_1 I_2}{2\pi d}$$

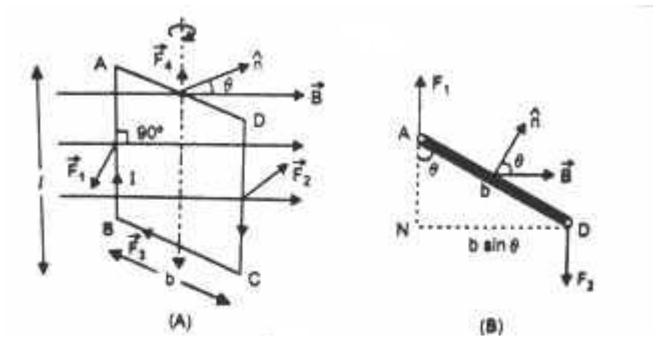
If $I_1 = I_2 = 1$ ampere and d=1m, then $f/l = 2x10^{-7} Nm^{-1}$.

Note:

Force between two parallel moving charges is given by

$F = \frac{\mu_0}{4\pi} \frac{q_1 q_2}{r^2} V_1 V_2$ where V_1 and V_2 are the velocities of two charges.

- ***Torque acting on a current loop placed in uniform magnetic field.***



Consider a rectangular loop **PQRS** of length **l** and breath **b** placed in a uniform magnetic field of strength **B**. let **I** be the current in the loop in clockwise direction as shown in figure.

Now the arms PS and QR will Experience equal and opposite force F, when the magnetic field is B.

$F=BIl\sin\theta$ ($\theta=90^0$)

$F=BIl$

Arm PQ and SR do not experience any force (PQ and SR are parallel to the field)

∴ torque = magnitude of either force Perpendicular distance between the forces

∴ Torque= F x QN

∴ $\tau=BIl \times b\sin\theta$

∴ $\tau=BI(l \times b)\sin\theta$

∴ $\tau=BIA\sin\theta$ ($l\times b=A\rightarrow$ area of loop)

∴ $\tau=MB\sin\theta$ ($IA=M\rightarrow$ magnetic dipole moment)

For N turns of loop

∴ $\tau=NIAB\sin\theta$

- **Note:**

1. If the normal of the plane of loop makes an angle α with magnetic field B.
Then $\theta+\alpha=90^0$ or $\theta=(90-\alpha)$

∴ $\tau=NIAB\sin(90-\alpha)$

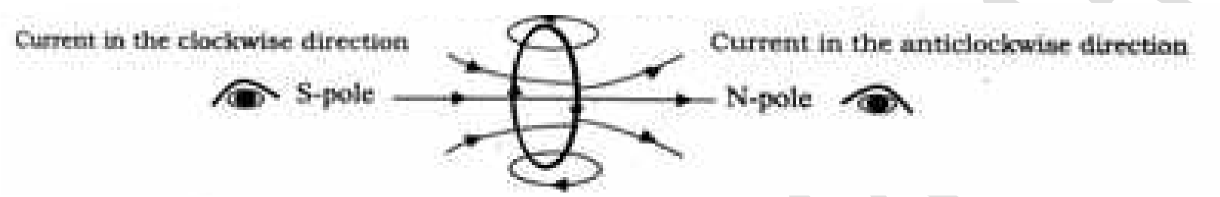
∴ $\tau=NIAB\cos\alpha$

1. Vector form of torque is $\vec{\tau}=\vec{M}\times\vec{B}$

2. The torque is maximum, when $\theta=90^0 (\tau=MB)$
3. The torque is minimum, when $\theta=0^0 (\tau=0)$

- ***Current loop behaves as a magnetic dipole:***

Magnetic field is produced round a circular loop when current is passed through it. The magnetic field is represented using magnetic field lines as shown in the figure. When the current in the loop is in clockwise direction, the face of the loop behaves as S-pole. When the current is in the anti-clockwise direction, the face of the loop behaves as N-pole. Hence, a current carrying loop behaves as a magnetic dipole



- **Regions why a current loop may be regarded as a magnetic dipole.**

1. It experiences a torque when placed it in an external magnetic field.

2. It generates its own magnetic field.

- ***Show that current loop behaves as a magnetic dipole.***

The electric field due to an electric dipole at a point on the axial line is given by,

$E=\frac{1}{4\pi\epsilon_o}\frac{2P}{r^3}$ -------------1

where P is the electric dipole moment and r is the distance between the dipole and the point of observation.

The magnetic field due to a circular loop at a point on the axial line is given by

$B=\frac{\mu_0}{4\pi}\frac{2\pi IR^2}{x^3}$

$B=\frac{\mu_0}{4\pi}\frac{2IA}{x^3}$ $\because \pi R^2$=A= area of the loop

$B=\frac{\mu_0}{4\pi}\frac{2M}{x^3}$ --------2

On comparing equation 1 and 2 , it is observed that the M represents the magnetic dipole moment and is given by M=IA

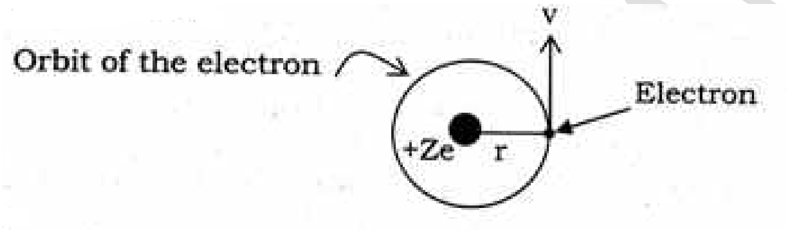
Hence, the current carrying loop behaves as a magnetic dipole.

Magnetic dipole moment of current loop: t is the product of current flowing through the loop and area of the loop. (M=IA).

- **Note:**
  1. Magnetic dipole moment of current carrying coil of n turns is M=nIA
  2. Its S.I. unit is Am^2. And its vector form is $\vec{M}$=I.$\vec{A}$

- ***Derive an expression for magnetic dipole moment of an electron revolving in the n^{th} orbit of an atom.***

In an atom, the evolution of electron around the nucleus is equivalent to current loop which is behaves as a magnetic dipole.



Let V-sped of the electron.

+Ze- charge on the nucleus.

R- Radius of the electron.

Let me be the mass of electron and e- charge on an electron.

The revolution of electron around the nucleus is equivalent to current loop which behaves as a magnetic dipole.

The magnetic dipole moment of this current loop is given by

M=IA------1

Current due to the electron, I=$\frac{e}{T}$-------2 $\because$ **I=q/t**

Where e is the charge on the electron and T is the period of revolution of electron.

Area of the loop, A=πr^2-------------3

Substitute equation 2 and 3 in 1 M=$\frac{e}{T}\pi r^2$ ----------4

We have, Time =$\frac{distance\ traveled}{speed}$

T =$\frac{2\pi r}{V}$------------5

M=$\frac{e}{2\pi r/V}$ πr^2

M=$\frac{eVr}{2}$

Divide and multiply by m_e to the LHS of above equation.

M=$\frac{m_e eVr}{2m_e}$

M=$\frac{e}{2m_e}(m_e Vr)$

But m_eVr=L→angular momentum of the electron.

M=$\frac{e}{2m_e}(L)$ ∵L=$\frac{nh}{2\pi}$ from Bhor`s quantization rule.

M=$\frac{e}{2m_e}(\frac{nh}{2\pi})$

M=$\frac{neh}{4\pi m_e}$ where n=1,2,3……….and h=Planck's constant.

Bhor`s magneton(M_B):

It is minimum value of magnetic dipole moment of an electron of hydrogen atom.

M_B=$\frac{eh}{4\pi m_e}$

Calculation of Bhor`s magneton:

M_B=$\frac{1.6x10^{-19}x6.625x10^{-34}}{4\pi x9.1x10^{-31}}$

M_B=9.27x10^{-24} Am^2.

Gyro magnetic ratio of an electron:

Gyro magnetic ratio =$\frac{magnetic\ dipole\ moment}{angular\ momentum}$=$\frac{M}{L}$

The moving coil Galvanometer (suspended coil Galvanometer)

A moving coil galvanometer is a sensitive instrument used for detecting small current (10^{-8} to 10^{-10} ampere current). The instrument is so named because it uses a current carrying coil that rotates in a magnetic field on account of torque acting on it.

- **Moving coil Galvanometer is of two types**

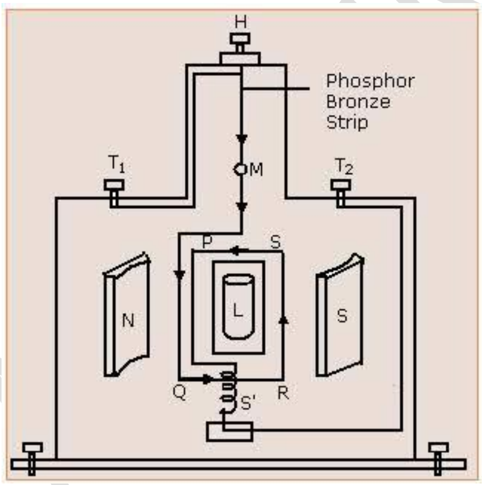
1. Suspended-coil galvanometer. 2. Pivoted-coil Galvanometer.

The principle and working of the two types of galvanometer are the same, only there is some difference in their constructions.

Principle:

It`s operation is based on the principle that when a current carrying coil is placed in a uniform magnetic field. The coil experiences a toque.

Construction:



It consist of a narrow rectangular coil PQRS consisting of a large number of turns of fine insulated copper wire wound over a frame made of light ,non magnetic metal. A soft iron cylinder known as the core is placed symmetrically within the coil and detached from it. The coil is suspended between the two poles of a permanent magnet which are cylindrical in shape. The coil is suspended with a movable torsion head H, by a strip or wire made of phosphor bronze which acts as path for the current to the coil also. End of the wire is connected to the terminal T_2 of the galvanometer. The other end of the coil is connected to light spring (S) which is finally connected to the terminal T_1. The spring exerts a very small restoring torque on the coil. A soft iron core is placed within the frame of the coil. A plane circular mirror (M) is attached on the wire or strip to note the deflection of the coil using lamp scale arrangement. The whole system is enclosed in a non-magnetic (wooden) case to avoid disturbance due to air.

- ***Radial magnetic field:***

The concave cylindrical pole pieces of the magnet and the soft iron cylinder used in the galvanometer. And makes the magnetic field radial and the field always remain parallel to the coil. Thus the limb of the coil always lots the magnetic field lines at right angles. The magnitude of the torque acting on the coil is same in all positions of the coil. This makes the scale of the instatement linear.

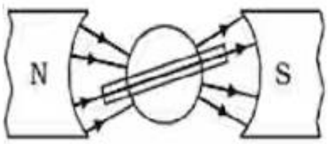
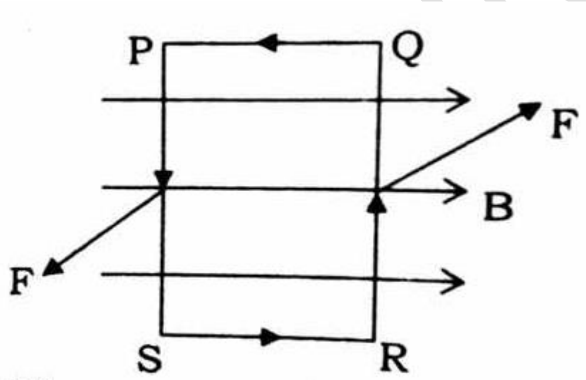


Fig Radial magnetic field

- ***Theory and working of moving coil galvanometer:***



In the above figure PQRS is a coil of galvanometer when current is passed through the galvanometer coil, it experiences torque due to the two forces acting on the faces PS &QR, it is called deflecting torque.

The deflecting torque is given by τ_D=BINA

Where B-magnetic field strength.

N-number of turns in the coil

A-area of the coil

The restoring torque due to the tension in the spring is given by

τ_R=Kθ where K-restoring torque per unit deflection and θ-deflection of the coil

The coil rotates until the restoring torque becomes equal to the deflecting torque.

Now the coil is in equilibrium positions

$\tau_D = \tau_R$ Kθ= BINA $I = \frac{K\theta}{NAB}$ I=Gθ

where G=K/NAB= constant for galvanometer

Iαθ

Hence deflection of the coil is directly proportional to the current through it.

- ***Sensitivity of moving coil galvanometer:***

A galvanometer is said to be sensitive if a small current passed through, it produces a large deflection. As we shall see, a galvanometer can be converted into an ammeter or voltmeter. **Accordingly it has two types of sensitivity viz.**

1. **Current sensitivity:** it is defined as the deflection produced in the galvanometer when a unit current passed through it.

   I.e current sensitivity $S_i = \frac{\theta}{I} = \frac{NAB}{K}$

   Unit is div/A

2. **Voltage sensitivity:** it is defined as the deflection produced in the galvanometer when a unit potential difference is applied across the two terminals of the galvanometer.

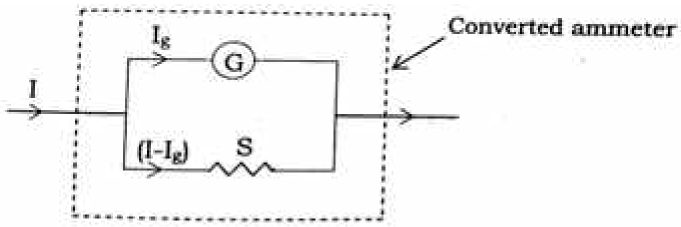
   i.e voltage sensitivity $S_v = \frac{\theta}{V} = \frac{\theta}{IR} = \frac{NAB}{KR}$

   Unit is div/V.

- ***Conversion of galvanometer into ammeter:***

A galvanometer can be converted into an ammeter by connecting a low resistance in parallel with it.

- **Calculation of low resistance (shunt) resistance(S):**



Consider a pointer galvanometer of resistance **G**. which shows full scale deflection for a current **Ig**. **S** is the low resistance called shunt connected in parallel with galvanometer to convert it into ammeter of range **0-I**. the value of **S** should be such that when a current of **I** enters the instrument only a small portion Ig flows through G and the remaining current **(I-Ig)** flows through **S.**

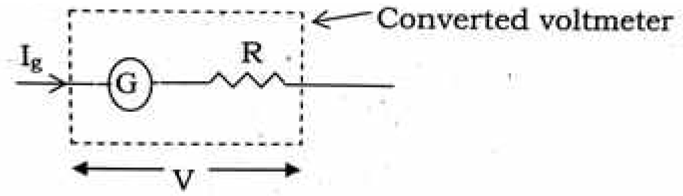
Potential difference across **S** = potential difference across **G**

$$(I-I_g)S = I_gG \qquad S = \frac{I_gG}{I-I_g}$$

- ***Conversion of galvanometer into voltmeter***

A galvanometer can be converted into a voltmeter by connecting high resistance in series with it.

- **Calculation of high resistance(R):**



Let R-be the high resistance connected in series with galvanometer.

G- be the resistance of the galvanometer.

Ig-be the current required for full scale deflection in the galvanometer.

V-be the maximum potential difference.

From ohms law

V=Ig(G+R)

G+R=$\frac{V}{I_g}$

R=$\frac{V}{I_g}$-G

- **Note:**

	Ammeter	Voltmeter
1	t is a device used to measure the current flowingin a circuit.	It is device used to measure the potential difference
2	It should be connected in series in the circuit.	It should be connected in parallel with resistance.
3	Its resistance should be very low	Its resistance should be very high.
4	Resistance of an ideal ammeter is zero	Resistance of an ideal voltmeter is infinity.

- **QUESTIONS FROM BOARD EXAMINATIONS**

One marks questions

1. What is the nature of force between two parallel conductors carrying current in the same direction.**[M-14]**
2. State amperes circuital law.**[J-14]**
3. A charged particle enters an electric field in the same direction of electric field. What is nature of the path traced by it?**[J-15]**
4. What is a cyclotron.**[M-16]**
5. When will the magnetic force on a moving charge e maximum in a magnetic field?**[J-16]**
6. What is Lorentz force.**[J-17]**
7. Give one application of cyclotron.**[M-18]**

8. Write the expression for force experienced by a straight conductor of length l carrying a steady current I moving in a uniform external magnetic field B.**[J-18]**

9. Write the expression for force acting on a moving charge in a magnetic field.**[M-19]**

10. When does the force acting on a charged particle moving in a uniform magnetic field is maximum.**[J-19]**

Two marks questions

1. Write any two uses of cyclotron.**[J-14]**

2. State ampere`s circuital law and represent in mathematically.[**J-14]**
3. What is toroid? Mention the expression for magnetic field at a point inside a toroid.**[M-16]**
4. A galvanometer having a coil of resistance 12Ω gives full scale deflection for a current of 4mA. How can it be converted into an voltmeter of range 0 to 24V.**[J-16]**
5. Write the expression for cyclotron frequency and explain the terms.**[J-18]**
6. Draw neat labelled diagram of cyclotron**[J-19]**
7. Mention the expression for magnetic field produced at the centre of the axis of current carrying solenoid and explain the terms.**[J-19]**
8. In a region, an electric field $\vec{E}$=5x10$^3\hat{j}$NC-1 and a magnetic field of $\vec{B}$=0.1$\hat{k}$ T are applied.A beam of charged particles are projected along X- direction. Find the velocity of charged particles which move undeflected in this crossed fields.**[M-20]**

Three marks questions.

1. Write any three uses of cyclotron.**[M-14]**
2. Give an expression for force acting on a charge moving in magnetic field and explain the symbols, when does the force becomes maximum**[J-14]**
3. Explain with circuit diagram how to convert galvanometer in to an ammeter.**[M-15,J-19]**
4. State ampere circuital law. Using it derive an expression for magnetic field at a point due to a long current carrying conductor.**[J-15,M-18]**
5. Explain with circuit diagram how to convert a galvanometer into a voltmeter.**[M-17,J-17]**
6. Write the expression for force per unit length between two straight parallel current carrying conductors of infinite length. Hence define SI unit of current "ampere"**[M-19]**
7. Give the principle of cyclotron and draw the neat lebelled diagram of cyclotron.**[M-20]**

Five marks questions.

1. Derive the expression for magnetic field at appoint on the axis of a circular current loop using Biot-savart law.**[M-14,M-15,M-16,M18,M-19]**
2. Derive the expression for the force between two parallel conductor carrying current and hence define ampere.**[J-15,M-16,J-16,J-17,J-18,M-20]**